VIBRATIONAL FLUIDIZATION OF A SHALLOW GRANULAR BED

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A model is given for a vibrationally fluidized bed that incorporates the density reduction and allows one to calculate the pressure difference across the bed averaged over the period of oscillation.

The physical parameters of a bed and of the medium in which it lies go with the geometrical features of the apparatus and the mode of vibration to produce a great variety of forms of behavior in a vibrationally fluidized bed [1], primarily because of the multiplicity of physical factors that can influence the fluidization, which considerably hinders any attempt at a detailed theory of the process.

It is inadequate to represent even a shallow granular bed used at a moderate vibrational frequency as a rigid porous system interacting with the grid and the medium, although this allows of comparatively simple analysis [2-4]. Dynamic features such as the expansion of the bed become very important [5, 6], which is related to the expansion occurring in an ordinary fluidized bed in response to the flow rate [7, 8], and these have a marked influence on the pressure fluctuations. This expansion is the main cause of the nonzero time-averaged* pressure difference across such a bed and the reduction in the hydraulic resistance caused by the vibration [5].

If the bed is relatively deep and the vibrational frequency is relatively high, the propagation of stress waves in the dense phase and of porosity waves in the expanded phase can begin to play a considerable part [9, 10], as can the compressibility of the fluid [11] and the wall friction [12]. Any analysis of these factors requires consideration of the viscoelastic and other parameters of the bed as a porous medium, and thus involves discussing the corresponding relaxation times and the wave propagation speeds, which means that in fact only the lower part of the bed vibrates in accordance with the physical model described in [6]; it is also evident that there are cohesion effects in a finely divided bed, which can alter the effective characteristics [13].

Here we neglect all effects concerned with nonuniformity in the state of stress, compressibility in the fluid (gas), and wall friction; i.e., we consider only a comparatively shallow bed. The hydraulic resistance of the bed and of the vibrating grid are assumed to be linear in the gas speed, while the motion of the bed is taken as one-dimensional.

Immediately after the bed becomes detached from the grid at time t_1, it moves upwards relative to the latter and exerts a resistance on the passing gas, which tends to accumulate in the increasing gap between the bottom of the bed and the grid. The hydraulic force acting between the particles within the bed and the gas flow is

$$f = CK(\rho)u, \quad K(0) = 1, \quad \rho = 1 - \varepsilon \tag{1}$$

and this is dependent on the local porosity ε and the relative gas speed u; the resistance coefficient C for small (Stokes) particles is $6\pi\mu a$, while numerous theoretical and empirical expressions are available for $K(\rho)$, which incorporates the effects of the hindered flow.

*To avoid misunderstanding, we must state at once that some workers, including [1], suppose that the pressure difference across the bed averaged over the period of vibration is an independent effect, while the static pressure difference is some quantity independent of the dynamic (pulsating) pressure. Physically speaking, this viewpoint is entirely erroneous, since the pressure is an intensive thermodynamic parameter. In fact, vibrational fluidization is highly nonlinear, since any given harmonic oscillation applied to this nonlinear system results in all multiple frequencies (as in any nonlinear system), including a component at zero frequency. A nonzero average pressure difference reflects the latter harmonic. This is emphasized by the use of the term average pressure difference instead of the inappropriate term static pressure difference.

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The hydraulic forces on particles at the upper and lower boundaries of the bed may differ from f, and in particular may be substantially less than the latter [14], so there may be no balance between the gravitational, inertial, and hydraulic forces for such particles; the sum of these forces is nonzero. It is then readily seen that the total force acting on a particle at the lower surface of a bed tends to return a particle to the latter when the bed begins to be detached from the grid (when the gap between the bed and the grid inceeases), i.e., the lower surface is stable in the same way as in the free surface of a fluidized bed. Conversely, the force acting on a particle at the upper surface tends to take the particle further from this surface in the incident gas flow; i.e., the upper surface is unstable in the sense that a particle leaving the latter moves faster than the center of gravity of the bed.

The set of such particles directly in contact with the incident flow may be identified with the effective upper boundary of the expanding bed; the relative motion of this boundary causes an increase in the porosity of the upper part of the bed. This results in a wave of elevated porosity, which propagates downward into the bed and is analogous to the waves that occur in ordinary fluidized beds in response to changes in fluid flow [7, 8].

In the final stage (when the gap between the bed and the grid decreases), the role of the boundary surfaces becomes different: The gas flow is then incident on a lower surface that approaches the grid more rapidly than does the center of gravity, while the upper surface becomes stable and the increased-porosity wave propagates from below upward. The bed clearly continues to expand.

Finally, at some instant t' the lower surface comes in contact with the grid, so the expansion of the bed ceases. The part of the bed that has fallen becomes closely packed, and the thickness of the close-packed part increases very rapidly with t. Let t'' be the instant at which the entire bed attains the close-packed state; i.e., this is the time when the upper surface of the bed falls and is less than the time $t_1 + 2\pi/\omega$ for fresh detachment of the bed from the grid, in which case the situation will repeat. If this is not so, the bottom close-packed part of the bed detached from the grid will collide with the descending particles in the fluidized state and will entrain the latter, so the mean porosity of the bed may even become less than that directly after detachment. In principle, piston structures may be formed, after several vibrational cycles, and therefore the nonuniformity of the bed, which previously was neglected, can become very substantial. Here we envisage only a very simple form of vibrational fluidization in which the bed has time to revert to the close-packed state as a whole before each fresh detachment.

The accelerated action of the boundary layer facing the incident gas flow can be described if we assume that the particles are acted on by a hydraulic force σf, where f is defined by (1) and $\sigma < 1$; unfortunately, no theoretical evidence is available on σ, while the experimental evidence [14] relates to single particles near a regularly packed bed and having a definite disposition with respect to the particles in the bed, although the measured forces are substantially dependent on the disposition. Therefore, the data of [14] cannot be used directly to estimate σ, which applies particularly to the particles at the boundary, since these are randomly disposed one with respect to another and with respect to the other particles in the bed. It is best to consider σ as an adjustable empirical parameter.

This σ and the various assumptions above make it meaningless to consider the porosity-wave propagation in excessive detail, which also is difficult even in a linearized formulation; on the other hand, it is physically obvious and well confirmed by experiment that for small times (usually on the order of 0.01-0.1 sec) and for $A/h_0 \ll 1$ the bed does not have time to expand substantially, and therefore the current height 2h and mean porosity ε differ only slightly from $2h_0$ and ε_0, the values in the close-packed state. The condition $\rho h = \rho_0 h_0 =$ const then gives us that

$$1-\varepsilon = \rho \approx \rho_0 (1-\alpha), \quad \alpha = (h-h_0)/h_0 \ll 1, \tag{2}$$

where the relative expansion α acts as a small parameter.

It is therefore natural to neglect the nonuniformity of the bed in the fluidized state, which is associated with porosity-wave propagation, and then the bed is considered as expanding uniformly, so we can utilize the mean porosity ε defined by (2). Here in most instances it is sufficient to assume that $\varepsilon = \varepsilon_0$ in order to calculate the hydraulic forces; the difference between ε and ε_0 becomes significant only in evaluating effects due solely to the expansion of the bed. Further, the smallness of α enables one to assume that the time interval $t''-t'$ is small by comparison with the fluidization phase $t''-t_1 \sim t'-t_1$.

We use the laboratory coordinate system x, in which the vertical coordinates of the vibrating grid and of the lower and upper boundaries of the bed and of the center of gravity are, respectively, $x_0 = A \sin\omega t$,

x_1, x_2 and $x_c = x_1 + h = x_2 - h$; we take $\varepsilon = \varepsilon_0$ to get the usual equation of motion for the center of gravity in terms of unit mass of particles [2-4]:

$$\ddot{x}_c = -g + \beta K Q, \; \beta = C/m, \; K = K(\rho_0), \tag{3}$$

where m is the mass of a single particle and a dot denotes differentiation with respect to time. In writing (3) we have neglected the difference between the hydraulic forces acting on the boundary particles and the f of (1), which is justified for $a/h_0 \ll 1$; we have also neglected the nonstationary components in the hydrodynamic interaction, which are important for $\omega \gg \mu/a^3 d_0$ [15]. Also, the relative infiltration rate is Q averaged over the bed, i.e., over x in the range from x_1 to x_2.

If the grid were impermeable, we would have $Q = -\dot{z}_c = -(\dot{x}_c - \dot{x}_0)$; on the other hand, $Q = -\dot{z}_c + q$, for a permeable grid, where q is the gas flow through unit area of the grid into the free cavity $x_0 < x < x_1$ $(0 < z < z_1)$. If the gas pressure above the bed is the same as that under the grid, while K' is the grid resistance coefficient, then we have within the working accuracy that

$$\Delta p = -2h_0\rho_0 d_1 \beta K Q = K'q, \; q = -\varkappa Q, \; \varkappa = \frac{2h_0\rho_0 d_1 \beta K}{K'} \tag{4}$$

and further

$$Q = -(1+\varkappa)^{-1}\dot{z}_c. \tag{5}$$

We transform (3) to a coordinate $z = x - x_0$ linked to the grid and use the obvious initial condition to get the following equation for z_c:

$$\ddot{z}_c + \frac{\beta K}{1+\varkappa}\dot{z}_c = -g + \omega^2 A \sin\omega t, \; t > t_1 = \frac{1}{\omega}\arcsin\frac{g}{\omega^2 A},$$
$$z_c = h_0, \; \dot{z}_c = 0 \; (t = t_1). \tag{6}$$

We now consider the motion of the upper boundary after detachment of the bed; the equation analogous to (3) takes the form

$$\ddot{x}_2 = -g + \sigma\beta K u_2, \; x_2 = x_c + h, \; u_2 = Q - \dot{h}. \tag{7}$$

Then in the z coordinate system we get

$$\ddot{z}_c + \ddot{h} = -g + \omega^2 A \sin\omega t - \sigma\beta K\,[(1+\varkappa)^{-1}\dot{z}_c + \dot{h}]. \tag{8}$$

We substract (6) from (8) to get the following equation for h:

$$\ddot{h} + \sigma\beta K\dot{h} = (1-\sigma)(1+\varkappa)^{-1}\beta K\dot{z}_c,$$
$$h = h_0, \; \dot{h} = 0 \quad (t = t_1). \tag{9}$$

The equation for the lower boundary at the end of the motion is entirely analogous to (8); the corresponding equation for h is

$$\ddot{h} + \sigma\beta K\dot{h} = -(1-\sigma)(1+\varkappa)^{-1}\beta K\dot{z}_c,$$
$$h = h_*, \; \dot{h} = \dot{h}_* \quad (t = t_*), \tag{10}$$

where h_* and $\dot{h}_*$ are the values of the function derived from (9) and the derivative at time t_*. The latter is the solution to t_* within the framework of our representation. Equations (9) and (10) are readily combined into one if we use $\dot{z}_c(t_*) = 0$ on the right in each; this applies up to time t' corresponding to fall of the lower boundary onto the grid.

If the hydraulic resistance of the grid is much less than that of the bed $(\varkappa \gg 1)$, the inhomogeneous parts in (9) and (10) essentially disappear, which corresponds to cessation of expansion in the detached bed; it is readily seen that in general the expansion of the bed increases monotonically as $\varkappa$ decreases, i.e., as the resistance of the grid increases, and it is maximal for $\varkappa = 0$, i.e., for an impermeable grid. Similarly, $\varkappa$ governs all the other effects related to expansion of the bed, in particular the nonzero average pressure difference. It has frequently been observed by experiment that this pressure difference vanishes and the pumping action of the bed ceases when the hydraulic resistance of the grid is small. An attempt has been made [16] to explain this in

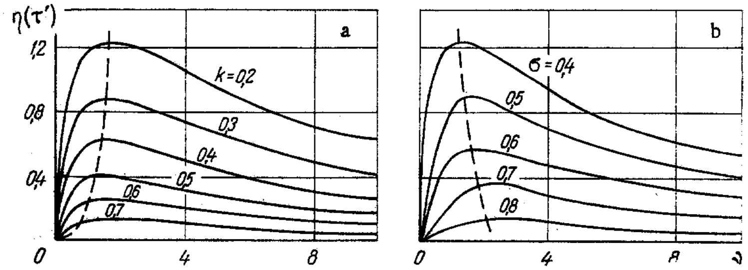

Fig. 1. Dependence of η at time τ' on ν for: a) σ = 0.6 and various k; b) k = 0.3 and various σ. The broken lines define the frequencies ν_m corresponding to maximum expansion of the bed.

terms of a valve effect at the perforated grid due to differences in occlusion of the apertures in the rising and falling states. The explanation was unsatisfactory because there is a free cavity above the grid in the rising phase, which is significant only for the production of the pressure difference across the bed, and this free cavity is almost entirely clear of particles. For definiteness, we consider only the case $\varkappa = 0$ in what follows; however, all the calculations can be readily carried through for any other value of $\varkappa$.

We introduce the following dimensionless variables and parameters:

$$\tau = \omega t,\ \zeta = \frac{z_c - h_0}{A},\quad \eta = \frac{h - h_0}{A},\ k = \frac{q}{\omega^2 A},\ \nu = \frac{\omega}{\beta K}. \tag{11}$$

Then from (6), (9), and (10) we get

$$\begin{gathered}\ddot{\zeta} + \nu^{-1}\dot{\zeta} = -k + \sin\tau,\quad \zeta = \dot{\zeta} = 0 \quad (\tau = \tau_1),\\ \ddot{\eta} + \sigma\nu^{-1}\dot{\eta} = (1-\sigma)\,\nu^{-1}|\dot{\zeta}|,\quad \eta = \dot{\eta} = 0\ (\tau = \tau_1).\end{gathered} \tag{12}$$

Here a dot denotes differentiation with respect to τ. The first equation in (12) is one first considered by Kroll [2, 3], and the solution is

$$\begin{gathered}\zeta(\tau) = -\frac{\nu^3}{1+\nu^2}(k\nu + \sqrt{1-k^2})\exp\left(-\frac{\tau-\tau_1}{\nu}\right) + \nu(k\nu + \sqrt{1-k^2}) -\\ - k\nu(\tau-\tau_1) - \frac{\nu^2}{1+\nu^2}\sin\tau - \frac{\nu}{1+\nu^2}\cos\tau.\end{gathered} \tag{13}$$

The solution to the second equation in (12) for the region $\tau_1 \le \tau \le \tau_*$ is

$$\begin{gathered}\eta(\tau) = \eta^0(\tau) - \eta^0(\tau_1)\exp\left[-\frac{\sigma}{\nu}(\tau-\tau_1)\right],\quad \eta^0(\tau) = \frac{1-\sigma}{\sigma}\nu\left[\frac{k\nu}{\sigma} +\right.\\ \left. + (k\nu + \sqrt{1-k^2})\right] - \frac{1-\sigma}{\sigma}k\nu(\tau-\tau_1) + \frac{\nu^3}{1+\nu^2}(k\nu + \sqrt{1-k^2}) \times\\ \times \exp\left(-\frac{\tau-\tau_1}{\nu}\right) - \frac{\nu^2}{1+\nu^2}\cdot\frac{1-\sigma^2}{\sigma^2+\nu^2}\sin\tau + \frac{\nu}{1+\nu^2}\cdot\frac{(1-\sigma)(\nu^2-\sigma)}{\sigma^2+\nu^2}\cos\tau\end{gathered} \tag{14}$$

and for the region $\tau_* \le \tau \le \tau'$, is

$$\begin{gathered}\eta(\tau) = -\eta^0(\tau) + \left\{2\eta^0(\tau_*) - 2\frac{1-\sigma}{\sigma}\zeta(\tau_*) - \eta^0(\tau_1)\exp\left[-\frac{\sigma}{\nu}(\tau_* - \tau_1)\right]\right\} \times\\ \times\exp\left[-\frac{\sigma}{\nu}(\tau-\tau_*)\right] + 2\,\frac{1-\sigma}{\sigma}\zeta(\tau_*).\end{gathered} \tag{15}$$

The instant τ_* is defined by $\dot{\zeta}(\tau_*) = 0$, which is readily written in explicit form by solving (13); therefore, the dimensionless paths of the center of gravity and of the bed expansion are dependent only on three dimensionless parameters: the above quantity σ, the quantity k (the reciprocal of the ordinary multiplicity of the vibrational acceleration), and the relative frequency ν. From (13)-(15) we readily derive simplified formulas corresponding to small k, small and large ν, and so on.

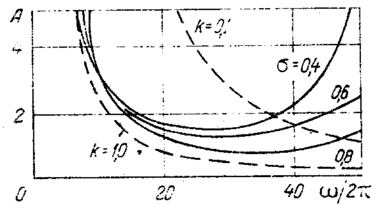

Fig. 2

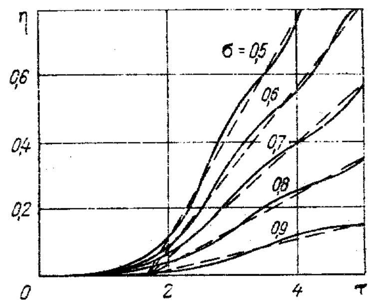

Fig. 3

Fig. 2. Curves in the parameter plane for vibrations that determine the maximum expansion of a bed of particles having $\beta K = 100$ $\sec^{-1}$; the numbers on the curves are the values of σ. The broken lines are lines of constant k. A, mm; $\omega/2\pi$, Hz.

Fig. 3. Dynamics of bed expansion for $k = 0.1$, $\nu = 1.0$ ($\tau_* = (3.8)$, and various σ.

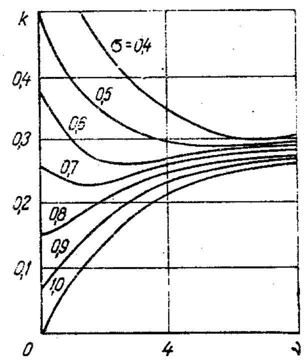

Fig. 4. Boundaries between mild vibrofluidization modes (not accompanied by collision of suspended layers of granular material) in the (k, ν) phase plane for various σ.

Strictly speaking, the solution to (13) applies only up to $\tau = \tau_2$, when the center of gravity of the bed is again at the level $z_c = h_0$, while the solution of (15) applies up to $\tau = \tau' < \tau_2$, when the lower boundary of the bed comes in contact with the grid. However, the time interval (dimensionless) $\tau'' - \tau'$ is small by comparison with $\tau_2 - \tau_1$, so (15) can still be used for η in the interval (τ'', τ'). The equations for these dimensionless times are

$$\begin{gathered} \tau_1 = \arcsin k, \ \dot{\zeta}(\tau_*) = 0, \quad \zeta(\tau') - \eta(\tau') = 0, \\ \zeta(\tau_2) = 0, \quad \zeta(\tau'') + \eta(\tau'') = 0. \end{gathered} \tag{16}$$

The conditions for realization of these modes (without collision between the beds of granular material in the suspended state) can then be put as

$$\tau'' < \tau_1 + 2\pi = \arcsin k + 2\pi. \tag{17}$$

Figure 1 shows the dimensionless expansion of the bed at time τ' as a function of ν for various k and σ; the expansion is maximal for the value ν_m of the dimensionless frequency, which is independent on k and σ. As would be expected, ν_m decreases monotonically as k increases for a given σ, while it increases monotonically with σ for a given k. The value of β in (3) is inversely related to the density and particle size, and directly to the density and viscosity of the gas, while the converse applies to the ν of (11). Therefore, this variation in the physical parameters should reduce the expansion of the bed for $\nu > \nu_m$; however, the converse effect should occur for sufficiently small dimensionless frequencies ($\nu < \nu_m$).

The $\nu = \nu_m(k, \sigma)$ curves are readily derived from curves such as those shown in Fig. 1, and they correspond to the maximum expansion in the (A, ω) parametric plane. Figure 2 shows such curves for various σ and $\beta K = 100\ \mathrm{sec}^{-1}$. These parametric relationships are of interest because in principle they allow one to define optimal values for the vibration parameters such that all effects due to bed expansion will be the most extensive. In particular, in some instances one naturally expects accentuation of the small-scale motion within the bed as the expansion increases, and hence acceleration of various transfer processes associated with such motion. Figure 2 shows also the lines of $k(A, \omega)$ = const for comparison.

Figure 3 shows a particular example of an expanding bed; $\eta(\tau)$ oscillates slightly around the asymptotic lines after a short interval, and all these lines emerge from a single point on the τ axis. Analytical equations are readily derived for these asymptotes, and these may be of value in approximate calculations.

Figure 4 shows the bounds to the existence of these states, as indicated by (17), which are realized if the image point in the phase plane lies above the boundaries. It is readily seen that any change in σ results in a substantial change in the curves in Fig. 4; on the other hand, the curves reach a steady level at high frequencies. The beds of granular material collide while suspended if the image point lies below the boundaries in Fig. 4.

We now consider the variations in pressure difference Δp in the bed (positive Δp correspond to reduced pressure in the gap between the bed and the grid). If we neglect the expansion of the bed for $\varkappa = 0$, we get from (4), (5), and (11) that

$$\Delta p = P\dot{\zeta}, \quad P = 2d_1\beta K\rho_0 h_0 A \tag{18}$$

which is the result implied by Kroll's theory [2, 3]. The actual dependence of Δp on A and βK is determined by the physical properties of the particles and gas, and it is nonlinear because these quantities are dependent on k and ν, which influence the dimensionless velocity ζ of the center of gravity. However, Δp is linearly dependent on h_0; moreover, it is readily seen that the pressure distribution over the depth of the bed at any time [apart from the short interval (t', t")] is also linear. The latter corresponds with experiment for sufficiently shallow beds, which are the ones to which this theory applies. The characteristic relationships between the quantity of (18) and the various parameters are familiar and need not be given here.

We see from (18) that the pressure under the bed is reduced in the initial stage ($\dot{\zeta} > 0$), whereas it is elevated by comparison with the pressure above the bed in the final stage. The average pressure difference is determined by averaging Δp over a vibration period, and in this approximation we have

$$\delta p = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} \Delta p dt = \frac{P}{2\pi}\int_0^{2\pi}\dot{\zeta}\, d\tau = \frac{P}{2\pi}\int_{\tau_1}^{\tau_2}\dot{\zeta} d\tau \equiv 0, \tag{19}$$

since $\zeta(\tau_2) = \zeta(\tau_1) = 0$ by definition.

Consequently, we may estimate the average pressure difference only if we incorporate the difference between ε and ε_0, i.e., use (6), (9), and (10) and the equations of (12) implied by these for $K(\rho)$ instead of $K = K(\rho_0)$; we use (2) with the obvious relation

$$\alpha = (A/h_0)\,\eta, \tag{20}$$

and find that the small-parameter method may be employed on the basis that

$$\zeta = \zeta^{(0)} + (A/h_0)\,\zeta^{(1)} + \ldots, \quad \eta = \eta^{(0)} + (A/h_0)\,\eta^{(1)} + \ldots, \tag{21}$$

where the quantities $\zeta^{(i)}$ and $\eta^{(i)}$ are of identical order in A/h_0; the equations for these are derived by means of standard perturbation theory. The equations for $\zeta^{(0)}$ and $\eta^{(0)}$ coincide with those written above in (12); i.e., these quantities are represented in the form of (13)-(15). The formulation and solution of linear equations for $\zeta^{(1)}$ and $\eta^{(1)}$ is elementary, but there is no need to do this in order to calculate δp. In fact, the deviations of the actual porosity from ε_0 are small, so instead of (8) we have up to terms of the first order in A/h_0 that

$$\Delta p = P\frac{K(\rho)}{K}\dot{\zeta} = P\left(1 - \frac{A}{h_0}N\eta\right)\dot{\zeta} = P\left(\dot{\zeta} - \frac{A}{h_0}N\eta^{(0)}\dot{\zeta}^{(0)}\right), \; N = \frac{\rho_0}{K}\frac{dK(\rho)}{d\rho}\bigg|_{\rho=\rho_0}. \tag{22}$$

Here we have used (2), (20), and (21), together with the identity $\rho h = \rho_0 h_0$, which reflects the conservation of the granular material.

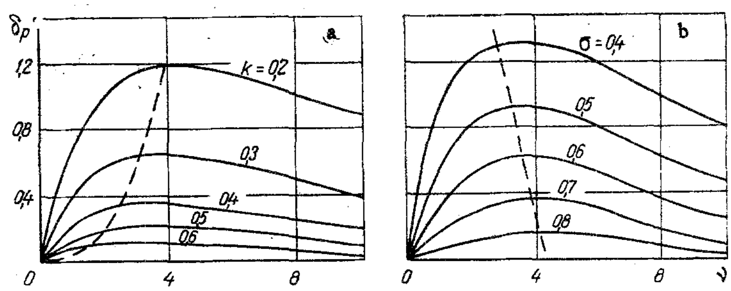

Fig. 5. Dependence of $\delta p'$ on ν for: a) $\sigma = 0.6$ and various k; b) $k = 0.3$ and various σ.

We average (22) over the period of oscillation in accordance with (19) and use the definitions of P and N of (18) and (22) to get within our accuracy that

$$\delta p = -\frac{APN}{2\pi h_0}\int_{\tau_1}^{\tau_2} \eta^{(0)}\dot{\zeta}^{(0)}\,d\tau = \left(\frac{\rho_0^2}{\pi}\cdot\frac{dK(\rho)}{d\rho}\bigg|_{\rho=\rho_0}\right) d_1\beta A^2 \delta p',$$
$$\delta p' = -\int_{\tau_1}^{\tau_2} \eta\dot{\zeta}\,d\tau. \quad (23)$$

As $K(\rho)$ is an increasing function, the sign of δp is determined by the sign of the integral in (23); a qualitative examination shows that $\delta p'$ is positive, at least for the states most commonly used, i.e., corresponds to a mean pressure reduction under the bed. An interesting point is that δp in that case is independent of the bed depth. Figure 5 shows the dependence of $\delta p'$ on the dimensionless parameters; the relationship resembles that for the dimensionless relative expansion of the bed in Fig. 1.

It would be quite possible to calculate the components of second and higher orders in the expressions for ζ, η, δp, and so on; however, we have neglected the nonuniformity of the expansion above, and therefore such calculations would exceed the accuracy of the physical formulation. Moreover, even the result of (23) is to be treated as approximate and applying only within a coefficient of the order of one.

The literature contains a great variety of suggestions on ways of classifying modes of vibrational fluidization; leaving aside the case of a vibroviscous bed ($k > 1$), we see that it is best to distinguish first shallow and deep beds, in which the effects of viscoelastic and other waves and of wall friction are correspondingly slight and substantial. Second, it is desirable to distinguish mild and severe vibrational fluidization states. In the first, the bed has time to sink down onto the grid before the next detachment, whereas in the second this is not so, and one gets colliding layers of fluidized material. On this basis, we have considered mild fluidization states in the above.

NOTATION

A	is the vibration amplitude;
a	is the particle radius;
C	is the hydraulic resistance coefficient for a particle;
d_0, d_1	are the gas and particle densities;
f	is the hydraulic force;
g	is the acceleration due to gravity;
h	is the half height of the bed;
K	is the function in (1) for constrained flow around particles;
K'	is the hydraulic resistance coefficient of grid;
k	is the reciprocal of vibrational-acceleration factor;
m	is the mass of a particle;
N	is the parameter in (22);
P	is the coefficient in (18);
Δp, δp	are the instantaneous and mean pressure drops in bed;
Q	is the speed relative to center of gravity;
q	is the gas flow rate through the grid;

t is the time;
u is the local relative speed;
x, z are the laboratory and grid vertical coordinates;
α is the relative expansion;
β is the reduced resistance coefficient in (3);
ε is the porosity;
η is the dimensionless relative expansion;
$\varkappa$ is the hydraulic resistance ratio (bed to grid);
μ is the viscosity;
ν is the dimensionless vibration frequency;
ξ is the dimensionless relative coordinate for the center of gravity;
ρ is the volume concentration of particles;
σ is the coefficient for boundary resistance reduction;
τ is the dimensionless time;
ω is the circular vibration frequency.

Indices

0 is the packed state;
1, 2, and c are the lower and upper boundaries and center of gravity, respectively;
* are the values when the center of gravity is at the maximum distance from the grid.

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